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MAXIMUM SPECIFIC POWER OUTPUT OF AN IRREVERSIBLE RADIANT HEAT ENGINE

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Abstract—With the help of an irreversible Carnot cycle model with continuous flow, the effects of the irreversibility of finite rate heat transfer and internal irreversibility of the working fluid on the performance of a radiant heat engine are studied. The specific power output of the heat engine is chosen to be an objective function for heat engine optimization. The maximum specific power output and the corresponding efficiency are derived. The optimal problems concerning the primary performance parameters of the heat engine, such as the efficiency, temperatures of the working fluid and heat transfer areas are discussed in detail.

Specific power   Irreversible heat engine

NOMENCLATURE

A = Total heat transfer area of heat engine (m^2)
A_1 = Heat transfer area between heat engine and heat source (m^2)
A_2 = Heat transfer area between heat engine and heat sink (m^2)
F_{CL} = Shape factor based on A_2
F_{WH} = Shape factor based on A_1
I = Internal irreversibility parameter of working fluid, defined in equation (5)
L = Distance from heat engine to sun (m)
p = Specific power output (W/m^2)
p_{max} = Maximum specific power output (W/m^2)
q_1 = Rate of heat transfer from heat source to heat engine (W)
q_2 = Rate of heat transfer from heat engine to heat sink (W)
R = Radius of heat source (m)
ΔS_1 = Entropy difference of working fluid for high-temperature isothermal process (J/K)
ΔS_2 = Entropy difference of working fluid for low-temperature isothermal process (J/K)
T_H = Temperature of heat source (K)
T_L = Temperature of heat sink (K)
T_1 = Temperature of working fluid in high-temperature isothermal process (K)
T_2 = Temperature of working fluid in low-temperature isothermal process (K)
U_1 = Heat transfer coefficient between heat engine and heat source [W/(K(m^2))]
U_2 = Heat transfer coefficient between heat engine and heat sink [W/(K(m^2))]
η = Efficiency
$\eta_{max,I}$ = Maximum efficiency
σ = Stefan–Boltzmann constant W/[K^4(m^2)]

INTRODUCTION

The optimum operation of radiant heat engines in finite time conditioning has been a subject of recent discussion [1–7]. Endoreversible Carnot cycle models have been employed to find the efficiency of radiant heat engines at maximum power output. On the basis of endoreversible Carnot cycle models, an irreversible Carnot cycle model with continuous flow will be established. The cycle model is used to analyze the performance of a radiant heat engine affected by the irreversibility of finite-rate heat transfer. Irreversible heat transfer occurs between the heat engine

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and its external heat reservoirs, as well as the internal irreversibility of the working fluid. The main purpose of this paper is not only to find the efficiency of the irreversible radiant heat engine at maximum specific power output but also to optimize other primary performance parameters of the heat engine.

THE CYCLE MODEL OF AN IRREVERSIBLE RADIANT HEAT ENGINE

Consider an irreversible radiant heat engine operating between a heat source at a high temperature T_H and a heat sink at a low temperature T_L. The cycle of the working fluid with continuous flow [8] consists of two isothermal and two adiabatic processes. The heat addition and rejection processes are assumed to take place simultaneously. Since thermal resistances between the heat engine and its external heat reservoirs exist, the heat transfer is carried out under a finite temperature difference so that the temperatures T_1 and T_2 of the working fluid in the heat addition and rejection processes are different from those of the two heat reservoirs. The radiation heat transfers are:

$$q_1 = U_1 A_1 (T_H^4 - T_1^4) \tag{1}$$

and

$$q_2 = U_2 A_2 (T_2^4 - T_L^4) \tag{2}$$

where q_1 and q_2 are the rates of heat transfer from the heat source at temperature T_H to the heat engine and from the heat engine to the heat sink at T_L; U_1 and A_1 are the heat transfer coefficient and area between the heat engine and the heat source; and U_2 and A_2 are the heat transfer coefficient and area between the heat engine and the heat sink, respectively. The total heat transfer area of the heat engine is

$$A = A_1 + A_2 \tag{3}$$

because it only exchanges heat with the two heat reservoirs at temperatures T_H and T_L.

Owing to the internal dissipations of the working fluid, all the processes of the cycle are irreversible. The entropy of the working fluid in the two adiabatic processes increases. The T–S diagram of the cycle is different from that of an endoreversible Carnot cycle [9] (as shown in Fig. 1), where ΔS_1 and ΔS_2 are, respectively, the entropy differences of the working fluid in the two isothermal processes at temperatures T_1 and T_2. By definition, these entropy changes are positive [10]. According to the second law of thermodynamics, one has

$$q_2/T_2 - q_1/T_1 > 0. \tag{4}$$

In order to obtain a quantitative relation among the parameters q_1, q_2, T_1 and T_2, an internal irreversibility parameter, I, is introduced.

$$I = \Delta S_2/\Delta S_1 \tag{5}$$

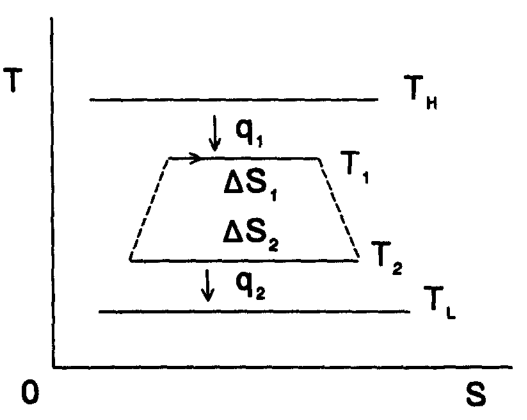

Fig. 1

This characterizes fully the degree of internal irreversibility resulting from the working fluid [11]. Using this parameter, the inequality in equation (4) can be written as

$$q_2/T_2 - I(q_1/T_1) = 0. \tag{6}$$

The cycle is endoreversible when $I = 1$. The cycle is internally irreversible if $I > 1$.

The cycle model described above is more realistic than the endoreversible cycle model of a Carnot heat engine. It includes both the irreversibility of finite-rate heat transfer and the internal irreversibility of the working fluid. These are the principal irreversibility sources of a class of radiant heat engines. In addition, it also includes the heat transfer areas which are important structural parameters of radiant heat engines. Therfore, the present cycle model can be used to optimize the structure of a radiant heat engine and to obtain the optimal performance of the heat engine.

MAXIMUM SPECIFIC POWER OUTPUT

Optimization of real heat engines designed by practicing engineers usually require a determination of the minimum heat exchanger area per unit net power output, or minimum cost per unit net power output. Thus, the specific power output [9, 12] (i.e. net power output per unit total heat transfer area) is chosen to be an objective function for heat engine optimization in this paper.

According to the above cycle model and equations (1), (2), (3) and (6), the specific power output of an endoreversible radiant heat engine is

$$p = (q_1 - q_2)A \tag{7}$$

$$p = \acute{\eta}\{1/(U_1(T_H^4 - T_1^4)) + (1 - \acute{\eta})/[(U_2/I^4)(T_1^4(1 - \acute{\eta})^4 - I^4T_L^4)]\} \tag{8}$$

where $\acute{\eta} = 1 - q_2/q_1 = 1 - I(T_2/T_1)$ is the efficiency of the heat engine.

For a given efficiency, maximizing p with respect to the temperature T_1 of the working fluid yields

$$\mathrm{d}p/\mathrm{d}T_1 = 0. \tag{9}$$

From equations (8) and (9), we find that the relations between the temperatures of the working fluid in the two isothermal processes and the efficiency of the heat engine are

$$T_1 = T_H[(c + b^4I^2/(1 - \acute{\eta})^{5/2})/(c + (1 - \acute{\eta})^{3/2}/I^2)]^{1/4} \tag{10}$$

and

$$T_2 = T_H[(c(1 - \acute{\eta})^4 + b^4I^2(1 - \acute{\eta})^{3/2})/(cI^4 + I^2(1 - \acute{\eta})^{3/2})]^{1/4} \tag{11}$$

where $c = (U_1/U_2)^{1/2}$ and $b = T_L/T_H$.

Substituting equation (10) into equation (8) yields

$$p = U_1T_H^4\acute{\eta}(1 - b^4I^4/(1 - \acute{\eta})^4/(1 + cI^2(1 - \acute{\eta})^{(-3/2)})^2. \tag{12}$$

The maximum specific power output of the heat engine can be obtained from equation (12) and plotted in Fig. 2. This plot shows that both the maximum specific power output p_{max} and the corresponding efficiency $\acute{\eta}_m$ decrease as I increases. It also shows that both the maximum specific power output p_{max} decreases as the efficiency decreases where $\acute{\eta} < \acute{\eta}_m$. In general, the heat engine should not be operated in the region of $\acute{\eta} < \acute{\eta}_m$. For an unlimited energy source, the heat engine should be designed to operate in the state of maximum specific power output, whereas for an energetically limited energy source, one has to reduce the specific power output so as to increase the efficiency of the heat engine. Therefore, the rational region of efficiency for an irreversible radiant heat engine is

$$\acute{\eta} > \acute{\eta}_m. \tag{13}$$

When the heat engine operates in the state of the maximum specific power output, the corresponding efficiency $\acute{\eta}_m$ can be directly derived from equation (12) and the external condition

$$\mathrm{d}p/\mathrm{d}\acute{\eta} = 0. \tag{14}$$

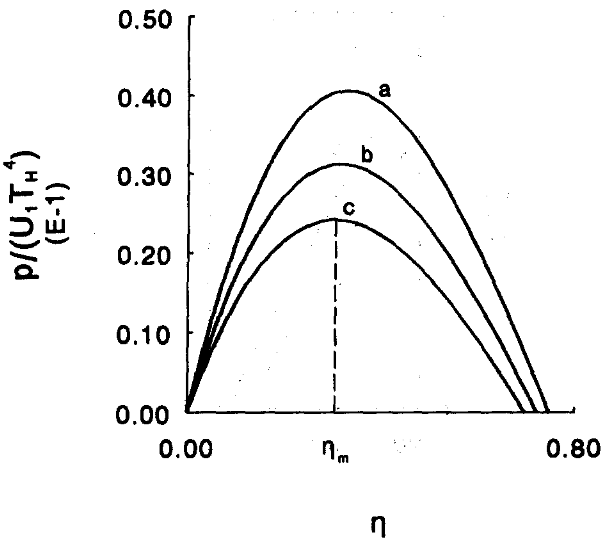

Fig. 2

The η_m is determined by the following equation:

$$(1-\eta)^{13/2}+4cI^2(1-\eta_m)^5-3cI^2(1-\eta_m)^4+3I^4b^4(1-\eta_m)^{5/2}-4I^4b^4(1-\eta_m)^{3/2}-cI^6b^4=0. \quad (15)$$

In principle, p_{max} and η_m can be calculated from equations (12) and (15) for the given parameters U_1, U_2, T_H, T_L and I.

OPTIMAL TEMPERATURES OF THE WORKING FLUIDS

Equations (10) and (11) indicate that the temperatures of the working fluid in the two isothermal processes are dependent on the efficiency. For example, when the efficiency attains its maximum at $T_1 = T_H$ and $T_2 = T_L$

$$\eta = 1 - I(T_L/T_H) = \eta_{max,I}. \quad (16)$$

This is a special case in which the effect of the thermal resistances may be neglected, and there still exists the internal irreversibility of the working fluid. In such a case, the specific power output of the heat engine is equal to zero. This implies that the efficiency of real radiant heat engines is smaller than $\eta_{max,I}$ because there always exists the internal irreversibility of the working fluid in real radiant heat engines.

At $T_1 = T_2$, the efficiency is equal to zero. The specific output power is also equal to zero. Obviously, real heat engines are not allowed to operate in such a state.

When $\eta = \eta_m$, the optimal temperatures of the working fluid in the two isothermal processes are

$$T_{(1,m)} = T_H[(c + b^4I^2/(1-\eta_m)^{5/2})/(c+(1-\eta_m)^{3/2}/I^2)]^{1/4} \quad (17)$$

and

$$T_{(2,m)} = T_H[(c(1-\eta_m)^4 + b^4I^2(1-\eta_m)^{3/2})/(cI^4 + I^2(1-\eta_m)^{3/2})]^{1/4}, \quad (18)$$

respectively. According to equations (10), (11) and (13), we can determine that the rational regions of the working fluid in the two isothermal processes should be

$$T_1 > T_{(1,m)} \quad (19)$$

and

$$T_2 < T_{(2,m)}, \quad (20)$$

respectively. The curves of T_1/T_H and T_2/T_L vs η are shown in Fig. 3.

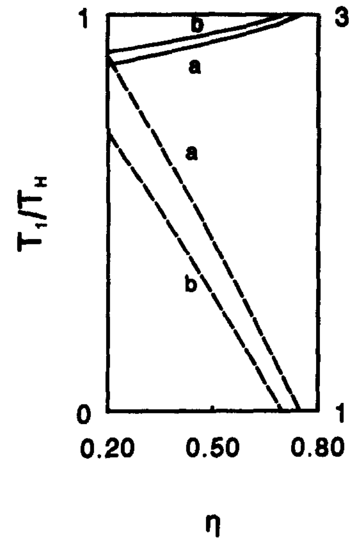

Fig. 3

OPTIMAL RATIO OF TWO HEAT TRANSFER AREAS

In order to make an irreversible radiant heat engine, operating in the optimal working states, the heat transfer areas A_1 and A_2 can not be chosen arbitrarily. They must satisfy a certain relation. From equations (1), (2) and (10), we find that the relation between A_1/A_2 and $\acute{\eta}$ is

$$A_1/A_2 = (1-\acute{\eta})^{3/2}/(cI^2). \tag{21}$$

When $\acute{\eta} = \acute{\eta}_m$, the optimal value of A_1/A_2 is determined by

$$(A_1/A_2)_m = (1-\acute{\eta}_m)^{3/2}/(cI^2)^{3/2}. \tag{22}$$

According to equations (21) and (13), the optimal ratio of A_1/A_2 should be

$$A_1/A_2 < (A_1/A_2)_m. \tag{23}$$

The curves of A_1/A_2 vs $\acute{\eta}$ are shown in Fig. 4.

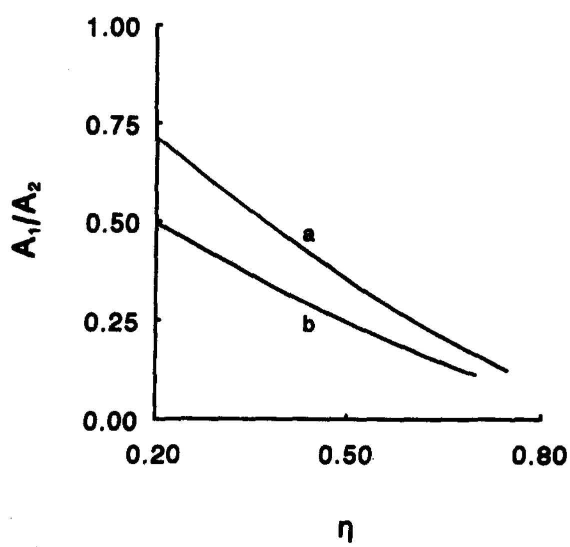

Fig. 4

Table 1. The values of the primary parameters of a solar radiant heat engine at maximum specific power output

I	η_m	p_{max}	$T_{(1,m)}$	$T_{(2,m)}$	$(A_1/A_2)_m$
1	0.8358	987.5	2911	478.1	14.27
1.1	0.8242	962.1	2972	475.0	13.06
1.2	0.8130	937.7	3028	472.0	12.04

NUMERICAL EXAMPLE

Consider a radiant heat engine which receives solar radiant heat from the sun ($T_H = 5755$ K) and emits radiant heat to space ($T_L = 0$ K) [4,6]. In such a case, the heat transfer coefficients are, respectively [6, 13],

$$U_1 = F_{WH}\sigma = \sigma R^2/(R^2 + L^2) \tag{24}$$

and

$$U_2 = F_{WC}\sigma \tag{25}$$

where σ is the Stefan–Boltzmann constant, $R = 6.98 \times 10^8$ m is the radius of the sum, $L = 1.49 \times 10^{11}$ m is the distance between the sun and the heat engine, and the shape factors are $F_{WH} = 2.176 \times 10^{-5}$ and $F_{CL} = 1$. The term $U_1 T_H^4 = F_{WH}\sigma T_H^4$ in equation (12) has a power density of 1353 W/m^2 which is the known solar constant. From the above equations, we can numerically calculate the values of the primary parameters of a solar radiant heat engine at maximum specific power output, which are listed in Table 1. It can be seen from Table 1 that, when $I = 1$, the efficiency of the heat engine and the temperatures of the working fluid are the same as the results of Refs [4] and [6]. It is important to note that the maximum specific power output of a radiant heat engine with continuous flow is two times as large as the results found in Refs [4] and [6].

CONCLUSION

We have demonstrated how an irreversible Carnot heat engine cycle model with continuous flow is established and used to analyze the performance of an irreversible radiant heat engine. Some important characteristics of the heat engine at maximum specific power output are found. The optimum values of the primary performance parameters, such as the efficiency, temperatures of the working fluid, and heat transfer areas, are determined. The results obtained here can guide the optimal design and operation of real radiant heat engines.

REFERENCES

1. A. DeVos, *Am. J. Phys.* **53,** 570 (1985).
2. A. DeVos, *J. Phys. D: Appl. Phys.* **20,** 232 (1987).
3. L. Chen and Z. Yan, *J. Chem. Phys.* **90,** 3740 (1989).
4. C. Wu, *Int. J. Ambient Energy* **10,** 145 (1989).
5. J. M. Gordon, *Am. J. Phys.* **58,** 370 (1990).
6. C. Wu, *Energy Convers. Mgmt* **33,** 279 (1992).
7. S. Gektun, S. Zkaynak and Y. Yavuz, *Energy* **18,** 651 (1993).
8. A. Bejan, *Advanced Engineering Theromdynamic.* Wiley, New York (1988).
9. C. Wu, *Energy Convers. Mgmt* **32,** 249 (1991).
10. J. Chen, *J. Phys. D: Appl. Phys.* **27,** in press.
11. C. Wu and R. L. Kiang, *Energy* **17,** 1173 (1992).
12. C. Wu, *Energy* **16,** 757 (1991).
13. F. Kreith, *Principles of Heat Transfer*, 3rd edn. Intext Educational, New York (1973).